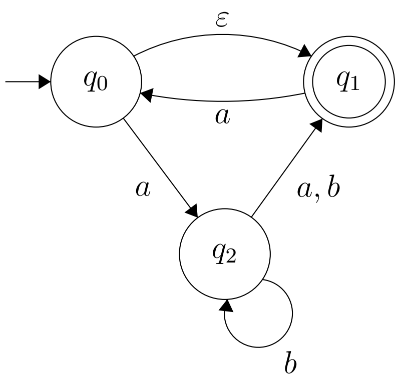
Exercise 1 - Computational Models - Fall 2017

1. For each of the following languages over $\Sigma = \{0, 1\}$, present a drawing representing a DFA that accepts it (correctness proof not needed):
   (a) Σ^*
   (b) $\{\varepsilon, 1, 01\}$
   (c) $\{\sigma w 0 \sigma \mid \sigma \in \Sigma, w \in \Sigma^*\}$
   (d) $\{w \mid w$ does not contain 0 or w ends with 01$\}$

2. Given n, let L_n be the language of words over $\Sigma = \{0, 1\}$ such that the nth character from the end is 0. Present a DFA that accepts L_n. Give a formal description and not a drawing.

3. (a) Give a formal description of the following NFA:



   (b) Convert the above NFA to a an equivalent DFA.

4. Present a regular expression for the following languages over $\Sigma = \{0, 1\}$:
   (a) $\{w|\ w$ contains exactly four '0's$\}$
   (b) The complement of $\mathcal{L}((1 \cup 01 \cup 001)^*(\varepsilon \cup 0 \cup 00))$

5. The following question deals with the equivalence between the two definitions for an NFA accepting a string given in class

Definition 1 *$N = (Q, \Sigma, \delta, S, F)$ accepts $w \in \Sigma^*$ if $\widehat{\delta}_N(S, w) \cap F \neq \emptyset$.*

Definition 2 *$N = (Q, \Sigma, \delta, S, F)$ accepts $w \in \Sigma^*$, if $\exists a = (a_1 a_2 \ldots a_k) \in (\Sigma_\varepsilon)^k$ and $r_0, \ldots, r_k \in Q$ s.t.,*

- $w = d(a)$[1]
- $r_0 \in S$
- $r_k \in F$
- $r_{i+1} \in \delta(r_i, a_{i+1})$, *for all* $0 \leq i < k$

Prove that if an NFA accepts a string according to definition 1 then it also does so according to definition 2

6. For languages A and B, let the *perfect shuffle* of A and B be the language

   $$\{w \mid w = a_1 b_1 \cdots a_k b_k, \text{ where } w_1 = (a_1 \cdots a_k) \in A \text{ and } w_2 = (b_1 \cdots b_k) \in B, \text{ each } a_i, b_i \in \Sigma\}$$

   Prove that regular languages are closed under this operation.

[1] For $a = (a_1 a_2 \ldots a_k) \in (\Sigma_\varepsilon)^k$, $d(a)$ is a without the ε symbols