Exercise 2 - Computational Models - Fall 2017

Note1: We denote by $\#_\sigma(w)$ the number of times the word $\sigma \in \Sigma^*$ is a substring in the word $w \in \Sigma^*$.

Note2: You may freely use results from the lectures and recitations.

1. Let L be a language over $\{0, 1, ..., 9\}$ such that $w \in L$ iff w is an i.d. number of an Israel citizen who like Kale.. Prove that L is regular.

2. Determine whether the following languages are regular. Prove your answer.

   (a) $L_1 = \{w \mid \#_a(w) \geq \#_b(w)\}$ over $\Sigma = \{a, b, c\}$.

   (b) $L_2 = \{w \mid |w| \in \mathbb{N}_{even} \ \wedge \ w = w^R\}$ over $\Sigma = \{0, 1\}$.

   (c) $L_3 = \{w \mid |w| \in \mathbb{N}_{even} \ \wedge \ w = w^R\}$ over $\Sigma = \{0\}$.

   (d) $L_4 = \{w \mid \exists n \in \mathbb{N} \ s.t. \ |w| = n^3\}$ over $\Sigma = \{1\}$.

3. (a) For each of the following, write a regular expression for $h(L)$:

      i. $L = L((00 \cup 1)*)$ and the homomorphism $h : \{0, 1\} \rightarrow \{a, b\}^*$ s.t. $h(0) = b, h(1) = \varepsilon$

      ii. $L = \{abab, baba\}$ and the homomorphism $h : \{a, b\} \rightarrow \{0, 1\}^*$ s.t. $h(a) = 01$ and $h(b) = 11$

   (b) For each of the following, write a regular expression for $h^{-1}(L)$:

      i. $L = L((00 \cup 1)*)$ and the homomorphism $h : \{a, b\} \rightarrow \{0, 1\}^*$ s.t. $h(a) = 01$ and $h(b) = 10$

      ii. $L = \{abab, baba\}$ and the homomorphism $h : \{0, 1\} \rightarrow \{a, b\}^*$, s.t. $h(0) = ab, h(1) = \varepsilon$

4. Give a representative for each equivalence classes of $\sim_L$ for the following languages:

   (a) $L_1 = \{w \mid |w| \ mod \ 3 \ = \ 0\}$ over $\Sigma = \{0, 1\}$.

   (b) $L_2 = \{0^m 1^n \mid m \neq n\}$ over $\Sigma = \{0, 1\}$.

5. Let $Inv(L) = \{xyz \mid xy^R z \in L\}$. Prove or disprove:

   - The regular languages are closed under this operation.

6. Reminder - $\Sigma^*/\backsim L$ is quotient set of the equivalence relation L. the set containing all the equivalence classes of Σ^* induced by $\backsim L$. Let $rank(L) = |\Sigma^*/\backsim L|$ - the number of equivalence classes induced by $\backsim L$. Prove or disprove:

   - Let L be a regular language. Then $rank(L) = rank(\overline{L})$
   - Let L_1, L_2 be a regular languages. Then $rank(L_1 \cap L_2) \leq rank(L_1) \cdot rank(L_2)$
   - Let L be a regular language over Σ and let $M = (Q, \Sigma, \delta, q_0, F)$ the DFA for L from the Myhill-Nerode theorem proof. Then, for every $N = (Q', \Sigma', \delta', S, F')$ a NFA such that $L(N) = L$, $|Q| \leq |Q'|$

7. This question deals with algorithmic problems.

   (a) Describe an algorithm that given a DFA A, decides if $L(A)$ is infinite. (possible hint: use the pumping lemma).

   (b) Describe an algorithm that given a DFA A, decides if $|L(A)| = 9,122,009$.