Exercise 3 - Computational Models - Fall 2017

1. Consider the grammar $G = (\{S, B\}, \{a, b, c\}, R, S)$ where R contains the rules

$$S \to abScB \mid \varepsilon$$
$$B \to bB \mid b$$

   What is $L(G)$? No need for a formal proof, but do provide an explanation.

2. Explain why the following grammar is ambiguous $G = (\{S, A, B\}, \{0, 1\}, R, S)$ where R contains the rules

$$S \to 0A \mid 1B$$
$$A \to 0AA \mid 1S \mid 1$$
$$B \to 1BB \mid 0S \mid 0$$

3. For each of the following languages, present a formal definition of a CFG (no need for a correctness proof, but do provide an explanation).

   (a) $L = \{0^i 1^j 2^k \mid i \neq j \text{ or } j \neq k,\ i, j, k > 0\}$ over $\Sigma = \{0, 1, 2\}$

   (b) $L = \{w \mid \#_0(w) = 2 \cdot \#_1(w)\}$ over $\Sigma = \{0, 1\}$

4. Prove using the Pumping Lemma that $L = \{a^n b^m c^n d^m \mid n, m \geq 0\}$ over $\Sigma = \{a, b, c, d\}$ is not context-free.

5. For each of the following languages present a diagram representing a PDA (no need for a correctness proof).

   (a) $L = \{w \mid \#_0(w) = \#_1(w)\}$ over $\Sigma = \{0, 1\}$

   (b) $L = \{a^i b^j | 0 \leq i \leq j \leq 2i\}$ over $\Sigma = \{a, b\}$

6. For languages A and B, let the *perfect shuffle* of A and B be the language

   $\{w \mid w = a_1 b_1 \cdots a_k b_k$, where $w_1 = (a_1 \cdots a_k) \in A$ and $w_2 = (b_1 \cdots b_k) \in B$, each $a_i, b_i \in \Sigma\}$

   Formally prove/disprove: If A and B are CFL then the *perfect shuffle* of A and B is also CFL.

7. Let L_r be any regular language over some alphabet Σ. Formally prove/disprove:

$$L' = \{u\sigma v \mid uv \in L_r \wedge \sigma \in \Sigma \wedge |u| = |v|\}$$

is context-free.