Exercise 5 - Computational Models - Fall 2017

The languages considered below are with respect to some fix alphabet Σ.

1. Let A, B and C languages over Σ. Prove/disprove:
   (a) If $A \leq_m B$ and $B \leq_m C$ then $A \leq_m C$.
   (b) If $A \leq_m B$ and $B \leq_m A$ then $A = B$.
   (c) If $A \subseteq B$ then $A \leq_m B$.
   (d) For every A and B, either $A \leq_m B$ or $B \leq_m A$.
   (e) If A is not trivial and context-free then $A \leq_m H_{TM}$.
2. For each of the following languages, find a minimal (relative to inclusion) class it belongs to (if any) between $\mathcal{R}/\mathcal{RE}/\text{co-}\mathcal{RE}/\text{not in } \mathcal{RE}\cup\text{co-}\mathcal{RE}$. Prove your answer.
   (a) $L = \{\langle M\rangle \mid \exists x \; s.t. \; M \text{ halt on } x\}$
   (b) $L = \{\langle M\rangle \mid L(M) \subseteq L(1(0 \cup 1)^*)\}$
   (c) $L = \{\langle M\rangle \mid L(M) = L(1(0 \cup 1)^*)\}$
   (d) $L = \{\langle M\rangle \mid M$ is a TM and there exists an input that the TM M accepts in less than 50 steps$\}$
   (e) $L = \{\langle M\rangle \mid |M$ is a TM and $L(M) \cup H_{TM} \in \mathcal{RE}\}$
3. Let CFG be the set of context-free languages. Prove/disprove:
   - For every $A \in CFG$, $L_A = \{\langle G\rangle \mid G$ is a CFG and $L(G) = A\} \in \mathcal{R}$
   - For every $\mathcal{C} \subsetneq CFG$ with $|\mathcal{C}| > 1$,
     $L_{\mathcal{C}} = \{\langle G\rangle \mid G$ is a CFG and $L(G) \in \mathcal{C}\} \notin \mathcal{R}$
4. Let $L = \{\langle M\rangle \mid M$ is a TM and $L(M)$ is Context Free$\}$.
   Show that $L \notin \mathcal{RE}\cup\text{co-}\mathcal{RE}$

5. For $L \subseteq \Sigma^*$ let $A(L) = \{0w \mid w \in L\} \cup \{1w \mid w \notin L\}$

   (a) Prove that if $L \notin R$ then $A(L) \notin \mathcal{RE}$

   (b) Prove or contradict: $\forall L \subseteq \Sigma^*$ if $L \leq_M \bar{L}$ then $L \in \mathcal{R}$
   Hint: use $A(L)$ as $L \notin \mathcal{R}$