Solution sketch 2 - Computational Models - Fall 2017

1. L is finite. Every finite language is regular.

2. (a) L_1 is not regular. Can be proved by Myhill-Nerode Theorem. Note that for every $m_1 \neq m_2$, $b^{m_1} \not\sim_{L_1} b^{m_2}$.

   (b) L_2 is not regular. Can be proved by the pumping lemma. For any $l \leq 0$ choose $s = 0^l 110^l$. Let $s = xyz$ s.t. $|y| > 0$ $|xy| \leq l$. Choose $i = 2$. $xy^i z \notin L$ (prove it).

   (c) L_3 is regular. Trivial.

   (d) L_4 is not regular. Use the pumping lemma. Almost similar to the proof for the same language with n^2 (done in the lecture).

3. (a) i. $(bb)^*$

      ii. $(01110111 \cup 11011101)$

   (b) i. $(ba)^*$

      ii. $1^*01^*01^*$

4. (a) $\{w \mid |w| \ mod\ 3 = 0\}$, $\{w \mid |w| \ mod\ 3 = 1\}$, $\{w \mid |w| \ mod\ 3 = 2\}$.

   (b) $\{0^m 1^n \mid m \underline{< n\}, \{}0^m\}$ for every $m \geq 0$, $\{0^m 0^k 11^k \mid k \geq 0\}$ for every $m \geq 1$, and $\mathcal{L}(0^*1^*)$.

5. (a) The regular languages are closed under this operation. Let $A = \langle Q, \Sigma, \delta, q_0, F \rangle$ be a DFA for L. Assume $Q = \{q_0, ..., q_n\}$. We build an NFA, A' for $Inv(L)$ as follows:

   - A' has 3 parts:
     - Part 1: A copy of A with no accepting states.
     - Part 2: For every $1 \leq i, j \leq n$ we will have an automata B_{ij} which is a copy of an automata for $Reverse(L)$ (how?).
     - Part 3: A copy of A.

- For every $1 \leq i, j \leq n$ we will have an ϵ transition from q_i in part 1 to q_j in B_{ij}, and an ϵ transition from q_i in B_{ij} to q_j in part 3.

Understand why does this gives an NFA for $Inv(L)$.

6. (a) Correct. let $M = (Q, \Sigma, \delta, q_0, F)$ the DFA with the minimal number of states that accept L. from Myhill-Nerode theorem $rank(L) = |Q|$. By replacing accepted with unaccepted states of M we get a DFA for $\overline{L}$ with $|Q|$ states. Thus the minimal number of states for DFA that accepts $\overline{L}$ is no more then $rank(L)$. This is why $rank(L) \leq rank(\overline{L})$. From the same claim for $\overline{L}$ we get $rank(\overline{L}) \leq rank(\overline{\overline{L}})$ and finally $rank(\overline{L}) = rank(L)$.

   (b) Correct. from Myhill-Nerode theorem exists M_1 and M_2 that accepts L_1 and L_2 such that $|Q_i| = rank(L_i)$. As seen in class we can build a product DFA from M_1 and M_2 with $|Q_1|*|Q_2|$ states that accept $L_1 \cap L_2$. Thus, in the minimal DFA for $L_1 \cap L_2$ there is no more then $|Q_1| * |Q_2|$ states. By Myhill-Nerode theorem $rank(L_1 \cap L_2) \leq rank(L_1) \cdot rank(L_2)$.

   (c) False. For the language $L = \{0\}$, the DFA with the minimal number of states that accepts L have 3 states. Thus, by Myhill-Nerode theorem $rank(L) = 3$. But, there is a NFA with 2 states that accepts L.

7. (a) **Claim:** Let A be a DFA with n states. $|L(A)|$ is infinite iff $\exists w \in L(A),\ n < |w| \leq 2n$.
   Proof: If $\exists w. |w| > n$ then as we learned in the pumping lemma, this word can be pumped infinitely and therefore $L(A)$ is infinite. if $L(A)$ is infinite, then there is a word $w \in L(A)$ such that $|w| > n$. The run of this word in A contains a cycle. We remove all cycles from the run and remember one simple cycle c, $|c| \leq n$. The run without the cycles give a word $w' \in L(A), |w'| < n$. We start pumping w' with the cycle c and we will eventually get a word in $L(A)$ in the proper length.
   This means that given a DFA A, we can run in A all the words w, such that $n < |w| \leq 2n$. If one of the words is accepted, then $L(A)$ is infinite, otherwise - finite.

   (b) First we check if $L(A)$ is infinite. If it is, we return $false$. otherwise, we run in A all words of length at most n and count how many are accepted. we return $true$ iff the count is 9,122,009.